It is considered that a combined use of the above described criteria gives a practical means to investigate the stability of complex tunnel-diode circuits and to determine the limits of permissible mismatch and diode characteristics variation.

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On "An Impedance Transformation Method for Finding the Load Impedance of a Two-Port Network"*

The above article[1] begins with ten extensive footnotes, but the authors omit the one reference that discusses their problem: Deschamps' "Hyperbolic Protractor."[2] The undersigned writers concede that the method of Mittra and King is different, but it is more complicated and less useful than that of Deschamps.

Concerning footnote 15, one notes that $R_{11}R_{22}-R_{12}^2 \geq 0$ is only a necessary but not sufficient condition for a positive definite quadratic form, and $R_{11}>0$, or $R_{22}>0$ is also required for sufficiency.

The undersigned find it somewhat surprising that many of the techniques found in Deschamps' pamphlet are not more widely used, for they apply to the interesting problems in measurements on linear passive reciprocal two-ports.

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* Received March 1, 1962.
[1] R. Mittra and R. J. King, IRE Trans. on Microwave Theory and Techniques, vol. MTT-10, pp. 13–19; January, 1962.
[2] G. A. Deschamps, "A Hyperbolic Protractor for Microwave Impedance Measurements and Other Purposes," Federal Telecommunication Labs., Nutley, N.J.; 1953. [See especially problems 7 (lossless case) and 13 (lossy case).]

Authors' Comment[3]

Concerning our paper,[1] Messrs. Stock and Kaplan have made the comment that our method for calculating an unknown load through a junction is complicated. To illustrate their point, they cite two examples (7 and 13) given in a booklet by Deschamps.[2,4]

We feel that Stock and Kaplan have missed the most important points in our paper. Section II of our paper is devoted to the establishment of a linear relationship between the input reflectance Γ_{in} and a modified load reflectance Γ_L' by $\Gamma_{in}-\bar{a}=\bar{b}(1-\Gamma_L')$. The constants $\bar{a}$ and $\bar{b}$ may be easily found but are not necessary for calibration. This linear relationship is the very heart of our paper, just as Deschamps' invariance of "hyperbolic" distances and "elliptic" angles is the basis of his booklet.

The examples chosen for comparison by Stock and Kaplan involve the so-called "three point method" which is subject to experimental errors. In any case, it is instructive to compare the two methods of solution of a typical example and let the reader decide which is more complicated and which is more accurate.

Example (*Problem 5—Lossless Case*): We have chosen problem 5 instead of 7 which is essentially the same but has additional property of more nearly showing the inverse transformation from load to input as well as the transformation from Γ_{in} to Γ_L'. The choice of the input reference is arbitrary so let us rotate the data given by Deschamps[2] clockwise 71° on the Smith chart so that when $\Gamma_L=+1(Z_L=\infty)$, $\Gamma_{in}=+1$. The data would then read as follows:

1) $Z_{in}=j90$ when $Z_L=0$,
2) $Z_{in}=\infty$ when $Z_L=\infty$,
3) $Z_{in}=81+j90$ when $Z_L=Z_{02}=200\ \Omega$.

What is the input impedance for a termination of $Z_L=520\ \Omega$?

Take $Z_{01}=100\ \Omega$ as the center of the input reflectance chart. It should be pointed out the original data given in the booklet has some error in that the input impedance corresponding to $Z_L=0$ should be $j12$ rather than $j10$ in order for the other two measurements to be consistent. This error may be fairly difficult to detect with the hyperbolic protractor because of the relatively large hyperbolic distances (measured in db) involved in this example.

Solution (*Mittra and King*): We have chosen the input reference such that the Γ_{in} plane coincides with the Γ_L' plane. Hence, using the transformation $z_L' \equiv r_L'+jx_L' = r_L/r_1+j(x_L+x_1)/r_1$ and data 3) above, $r_1=1/0.80=1.233$ and $x_1/r_1=0.90$ for $z_L=1$, which determines the calibration constants r_1 and x_1. To obtain $z_L'=z_{in}$ for $Z_L=520\ \Omega$ we again apply the transformation relating z_L' to z_L. ($r_L=2.60$, $x_L=0$.) $z_L'=z_{in}=r_L/r_1+jx_1/r_1=2.105+j0.90$ which completes the discussion. It is obvious that the inverse problem, *i.e.*, that of finding Z_L when Z_{in} is given is just reciprocal.

Solution (*Hyperbolic Protractor Method*): Since many readers do not have access to the booklet describing the use of the hyperbolic protractor we reproduce the solution in Fig. 1. Plot Q', P' and O' corresponding to data 1), 2) and 3) above. When the output port is matched (200 Ω), the corresponding input reflectance point is O' which is called the "iconocenter" by Deschamps. The transformation may be made to the projective chart by constructing $\bar{O}=\beta(O')$. Distances on the reflectance (Smith) charts are denoted by [ ] and by ⟨ ⟩ on the projective charts. Reflectances at the input port are denoted by primes and points on the projective chart by bars. Thus $\langle O\bar{O}\rangle = 2[O\bar{O}']$ = 17 db on Fig. 1. The points P' and Q' do not change in this transformation β and therefore the point $\bar{O}$ should fall on the straight line $\overline{QP}$, the image of the diameter QP. The point $\bar{W}$ which represents the input reflectance for 520 Ω at the output will be on $\overline{QP}$ at the hyperbolic distance $\langle \overline{OW}\rangle = [OW] = 8$ db or 16 db as measured on the projective chart. This immediately gives a means for constructing $\bar{W}$ which should be between $\bar{O}$ and P' since W itself lies between O and P. Measuring $\langle O\bar{W}\rangle$ with the protractor it is found to be 16 db, and W' is obtained by taking the hyperbolic midpoint of $\langle O\bar{W}\rangle$ or 8 db. The corresponding impedance obtained from the reflectance chart is then $2.06+j0.90$.

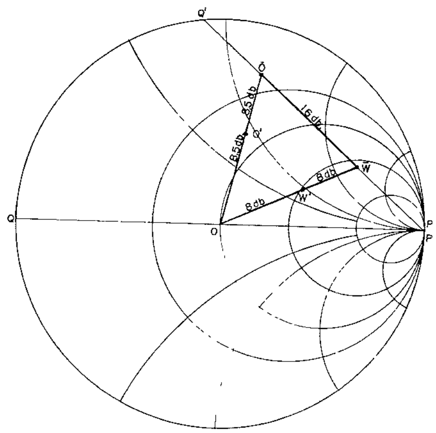

Fig 1.

Example (*Problem 13—Lossy Case*): The given data are:

1) Iconocenter = 8 db/−90°,
2) $\Gamma_{in}=9$ db/28° when $Z_L=0$.

What is the unknown load impedance when $\Gamma_{in}=10$ db/−134°?

Solution (*Mittra and King*): This example shows how one deals with the image circle rather than the unit circle but follows the same steps.

Expand the Γ_{in}-circle linearly and rotate it to correspond to the Γ_L'-circle. Read the transformed iconocenter from the Γ_L' plane as $z_{LO}'=0.625-j0.65=r_L/r_1+j(x_L+x_1)/r_1$. The load corresponding to this point is $z_L=1+j0$, so $r_1=1.60$ and $x_1=-1.04$. Now read the transformed point corresponding to the unknown load impedance as $z_L'=0.375-j0.043$. Using the impedance transformation equation as before we find $z_L=0.60+j0.991$. Thus, the actual load reflectance is $\Gamma_L=11.1$ db/80.5°. Incidentally, Deschamps' angle $(CP'', CL'')=80.5°$, not 71°, in problem 13.

[3] Received June 1, 1962.

[4] G. A. Deschamps, "A new chart for the solution of transmission line and polarization problems," IRE Trans. on Microwave Theory and Techniques, vol. MTT-1, pp. 5–13; March, 1953. This paper describes the theory discussed in Deschamps "Hyperbolic Protractor" and is included in that booklet.

Note that in either the lossy or lossless case a single arbitrary point inside the image circle is all that is necessary to completely calibrate the junction provided the corresponding Z_L is known.[5] However, a pure reactive termination is often more convenient and only two values are necessary (other than the point $x_L=\infty$ or $\Gamma_L=\Gamma_L'=+1$) as pointed out in Section III of our paper. Neither of these examples utilizes the technique of the cotangent method discussed in Section IV which is one of the most prominent features of the paper, particularly when dealing with lossy structures. It provides increased accuracy brought about by plotting a mean straight line from data corresponding to several reactances at the output port. Also note that in the lossless case of example 5, the calibration and solution are completely analytic when the original data are taken as absolute and the reference is chosen properly so that $Z_{in}=Z_L'$.

Stock and Kaplan also maintain that our method is less useful. Since they do not elaborate further we are unable to remark without specific examples. We do want to clearly state that although we do not claim our method to be a cure-all, the best, or any terms of such superlatives, we do believe it is a convenient, useful and accurate method of calibrating a junction.

Lastly, we acknowledge the typographical error and omission in our paper and agree that footnote 15 should read:

"The necessary and sufficient conditions for positive reality is that $R_{11}>0$, or $R_{22}>0$ and $R_{11}R_{22}-R_{12}^2>0\cdots$." We also agree that the "Hyperbolic Protractor" booklet should have been mentioned in our references.

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[5] The iconocenter is a special case and corresponds to $Z_L=Z_{02}$. Deschamps has gone to great length for devising methods of constructing this point.

Messrs. Stock and Kaplan's Reply[6]

We are most deeply indebted to Messrs. Mittra and King for their worked out examples comparing Deschamps' method with theirs, as well as for pointing out misprints in the hyperbolic protractor pamphlet. The object of our note was not to imply that the Mittra-King method should not have been published, but to point out that it is one among many equally valid and simple techniques for two-port calibration. Specifically, it is an alternate method to that of Deschamps for some problems. The choice of a given method is subjective, and while we realize the linearization technique applied to the determination of two-port input impedance and to the Weissfloch method is a contribution, we feel that the method of Deschamps is simpler in allowing one to work graphically with plotted data as would be obtained from a loss circle measurement. In fact, for determination of input impedance for a given load impedance of a two-port, the methods of de Buhr[7] or Bolinder,[8] seem simplest, involving a calculation of only an interative impedance plus a graphical construction. Indeed, the simplest calculation of two-port input-output relations involves only the use of the cross-ratio. For lossless networks, Bracewell's[9] nomograph is the simplest method of determining the input(output) impedance corresponding to a known output(input) impedance though the further extension by Hinckelmann[10] is not that simple.

In this connection, it may be pointed out that Deschamps' hyperbolic distance is only the logarithm of a cross-ratio, so may be calculated as accurately as one pleases from data. It should be further pointed out that the three-point method is not inherent in the geometrical technique; in fact, the virtue of the technique is that constructions may be made directly on a unit circle containing plotted data. It is not necessary to expand the loss circle once plotted.

We further note that in problem 5 as done by Mittra-King they do not correct their answer for the 71° phase shift. While reference planes are arbitrary for an illustrative problem, an actual network may have definite reference planes.

The iconocenter, contrary to footnote 4 of the Mittra-King rebuttal is simple to construct: one applies the butterfly construction to the intersection of chords connecting quarter-wave separated data points. Its great value is that it represents the transform of a perfect load, obtained without the necessity of having such a load.

There are a number of problems in which all the loss circle methods become increasingly cumbersome, *e.g.*, in the three- and four-ports much more work is needed here to devise convenient measurement methods. Stein[11] has illustrated well the magnitude of these problems.

[6] Received June 12, 1962.

[7] J. de Buhr, "Eine neue Methode zur Bearbeitung linearer Vierpole," *FTZ*, vol. 8, pt. I, pp. 200–204, April, 1955; pt. II, pp. 335–340, June, 1955.
——, "Die zeichnerische Bestimmung der geometrischen Kenngrossen verlustloser, linearer Vierpole," *AEU*, vol. 9, pp. 350–354; August, 1955.
——, "Die geometrische Darstellungsweise kombinierter linearer Vierpole," *AEU*, vol. 9, pp. 561–570, December, 1955.
——, "Die geometrische Darstellungsweise des Parallel- und des Serienblind-widerstandes als verlustfreie sogenannte parabolische Vierpole," *Nachrtech. Z.*, vol. 8, pp. 636–641; December, 1955.
——, "Die geometrische Vierpol-Darstellung des Doppeltransformators," *AEU*, vol. 10, pp. 45–49; January, 1956.
——, "Die geometrische elementarste Darstellungsform verlustloser linearer Vierpole," *Nachrtech. Z.*, vol. 9, pp. 80–84; February, 1956.
——, "Die geometrische Darstellungsweise hintereinander geschalteter allgemeiner, verlustbehafteter Vierpole," *AEU*, vol. 11, pp. 173–176, April, 1957.

[8] E. F. Bolinder, "Impedance and Power Transformation by the Isometric Circle Method and Non-Euclidean Hyperbolic Geometry," Radiation Lab., M.I.T., Cambridge, Rept. No. 312; June 14, 1957.

[9] R. N. Bracewell, "A new transducer diagram," Proc. IRE, vol. 42, pp. 1519–1521; October, 1954.

[10] O. Hinckelmann, "Graphical method for transforming impedances," IRE Trans. on Microwave Techniques, vol. MTT-10, pp. 139–141; March, 1962.

[11] S. Stein, "Graphical analysis of measurements on multi-port waveguide junctions," Proc. IRE (*Correspondence*), vol. 42, p. 599; March, 1954.

Messrs. Mittra and King[12]

There never has been any question in the mind of the present authors that Prof. Deschamps' geometric viewpoints are outstanding contributions to the theory of Microwave Measurements. They also agree with Kaplan and Stock that there are several methods which possibly provide simple graphical means for relating input and output impedance through a junction; although the word "simple" must be used in a subjective sense in relation to the different methods. We should like to point out again that the main emphasis in our paper is the averaging technique for smoothing out the errors.

Regarding their criticism of our workout of Problem 5, it should be pointed out that there was the implication in deriving the solution that the problem was stated after the reference plane shift, and hence no correction was necessary in the solution.

As regards the iconocenter, it is our experience that its determination for a particular set of experimental data is difficult when the experimental intersection points are distributed over a region due to the presence of experimental errors in the measurements. This is particularly true when the discontinuity being measured is small. We agree with Kaplan and Stock that the actual construction of the iconocenter procedure is straightforward if the exact values of Γ_{in} are known, but, of course, in practice such is not the case as finite errors are obviously present. Indeed, these were the very reasons why the Linear Transformation Method was developed.

We hope that we have made our position on the major issues sufficiently clear through the two communications and that this will end further discussions along similar lines.

[12] Received July 12, 1962.

On "A Solid-State Microwave Source from Reactance-Diode Harmonic Generators"*

In a recent paper,[1] Hyltin and Kotzebue have given some interesting formulas about the maximum efficiency of reactance-diode harmonic generators. It may be interesting to submit to the attention of the authors some considerations about their theoretical analysis.

1) Formula (11) and the consequent results were obtained by Hyltin and Kotzebue through a matrix inversion, starting from the hypothesis of a voltage controlled variable capacitor. It is also possible to arrive at the same result, more directly, starting from the dual case of a charge controlled variable elastance $S_d(q)$ (Fig. 1), which may be ex-

* Received March 1, 1962.

[1] T. M. Hyltin and K. L. Kotzebue, IRE Trans. on Microwave Theories and Techniques, vol. MTT-9, pp. 73–78, January, 1961.